

**The Department's Educational Philosophy**

The study of mathematics will enhance the ability of all students to problem solve and to reason. Through a strong standardized departmental program that emphasizes problem solving, communicating, reasoning and proof, making connections, and using representations, students will develop self-confidence and a positive attitude towards mathematics.

Our curriculum matches that of the Massachusetts Mathematics Curriculum Framework, and we are philosophically aligned with the National Council of Teachers of Mathematics Standards.

**Guiding Principles**

- Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.
- Effective mathematics programs focus on problem solving and require teachers who have a deep knowledge of the discipline.
- Technology is an essential tool in a mathematics education, and all students should gain facility in using it where advantageous.
- All students should have a high-quality mathematics program.
- Assessment of student learning in mathematics should take many forms to inform instruction and learning.
- All students should understand the basic structure of mathematics.
- All students should recognize that the techniques of mathematics are reflections of its theory and structure.
- All students should gain facility in applying mathematical skills and concepts.
- All students should understand the role of inductive and deductive reasoning in mathematic and real life situations.

## **PRECALCULUS/INTRODUCTION TO CALCULUS (H): COURSE #331**

**Course Frequency:** Full-year course; five times per week

**Credits Offered:** Five

**Prerequisites:** A final grade of at least 80 in Algebra II Honors or 90 in Algebra II AE

### **Background to the Curriculum**

This course uses the McDougal Littell text, Advanced Mathematics – Precalculus with Discrete Mathematics and Data Analysis, Brown, 2010 edition. It is the “Precalculus” course, which fits between the Algebra II program and one of the Advanced Placement Calculus classes. This text has been used since 1993; it replaced the text Advanced Mathematics by Coxford, published by Harcourt Brace Jovanovich, which had been used since 1980. The text is followed quite closely, and all material is covered except for the Data Analysis material. The text matches the 2000 edition of the Massachusetts State Frameworks for a Precalculus course and is philosophically aligned with the spirit of the National Council of Teachers of Mathematics Curriculum Standards. Teachers bring in other material where appropriate or time permits after consultation with the Regional Department Leader.

### **Core Topics / Questions / Concepts/ Skills**

Linear, Quadratic, Polynomial Functions

The General Theory of Functions

Trigonometric Functions and Their Inverses

Graphing Theory for Trigonometric Functions

Trigonometric Identities – Pythagorean, Sums, Difference, Double Angle, Half Angle

Polar Coordinates for Points and Complex Numbers

Vectors in Two Space and Three Space

Parametric Equations

Matrices in Equation Solving and Transformations

Limit Theory

Derivatives of Polynomials and Other Simple Functions

Applications of Derivatives

Exponential and Logarithmic Functions

Analytic Geometry of the Conic Sections

## **Course-End Learning Objectives**

*Students will be able to:*

- 1] Find the equation of a linear function, given various information.
- 2] Find the equation of a quadratic function, given various information.
- 3] Model real-world situations by linear or quadratic functions.
- 4] Use formulae for the distance between points and the distance from a point to a line.
- 5] Sketch linear and quadratic functions, given various information.
- 6] Use the Remainder and Factor Theorem.
- 7] Graph polynomial functions and determine an equation for a polynomial graph.
- 8] Write polynomial function for a given situation and find its maximum and/or minimum values.
- 9] Use technology to approximate the real roots of a polynomial equation.
- 10] Solve polynomial equations by various methods, including the Rational Root Theorem.
- 11] Apply general theorems about polynomial equations.
- 12] Solve linear, quadratic, and polynomial inequalities, including those with variable denominators.
- 13] Solve certain applied problems using linear programming.
- 14] Determine domain, range, zeroes of a function, and graph it.
- 15] Perform the composition and inverse of functions.
- 16] Determine periodicity and amplitude from graphs, stretch and shrink graphs vertically and horizontally, and translate them.
- 17] Find the measure of an angle in degrees or radians.
- 18] Find arc length and sector area in a circle.
- 19] Use the definitions of cosine and sine in terms of  $x$ ,  $y$ ,  $r$ .
- 20] Find values of trig functions using the definitions, reference arcs, special angles, and a calculator.
- 21] Find values of the tangent, cotangent, secant, and cosecant functions.
- 22] Find values of the inverse trig functions.
- 23] Graph the basic trig functions using concepts of domain and range.
- 24] Solve simple trig equations.
- 25] Find equations of sine and cosine curves that model periodic behavior.
- 26] Simplify trig expressions and prove trig identities.
- 27] Use trig identities or technology to solve more difficult trig equations.
- 28] Use right triangle trig, the Law of Sines, and the Law of Cosines to find unknown sides or angles of triangles.
- 29] Find areas of triangles using Trigonometry.
- 30] Derive and use identities for cosine, sine, and tangent involving sums, differences, double angles, and half angles.
- 31] Solve trig inequalities using identities or technology.
- 32] Use a variety of trig identities to simplify expressions and solve equations involving inverse trig functions.

- 33] Convert polar points and equations to Cartesian points and equations, and vice versa.
- 34] Graph the classical polar equations.
- 35] Convert complex numbers to polar form and use DeMoivre's Theorem.
- 36] Perform basic operations on vectors in 2 space and 3 space.
- 37] Use coordinates to perform vector operations in 2 space and 3 space.
- 38] Find norm and direction angle of a vector in 2 space.
- 39] Use vector and parametric equations to describe motion in the plane and in space.
- 40] Define and apply the dot product in 2 space and 3 space.
- 41] Use determinants to solve algebraic and geometric problems.
- 42] Define and apply the cross product of vectors in 3 space.
- 43] Find the equation of a plane.
- 44] Use sequences defined by general term or recursively.
- 45] Use arithmetic and geometric sequences and series.
- 46] Find or estimate the limit of an infinite sequence.
- 47] Find the sum of an infinite series.
- 48] Write proofs using the Principle of Mathematical Induction.
- 49] Find the sum, difference, product of matrices.
- 50] Find the inverse of matrices and solve linear systems using matrices.
- 51] Use technology to solve systems of equations using matrices and their inverses.
- 52] Find the images of sets of points under different types of transformations using matrices.
- 53] Use integral and rational exponents.
- 54] Use general exponential functions and the natural exponential function.
- 55] Define and apply logarithms and use log laws.
- 56] Solve exponential equations, and change logarithms from one base to another.
- 57] Understand the relationship between logarithmic and exponential functions.
- 58] Find the limit of a function as  $x \rightarrow \infty$ ,  $x \rightarrow -\infty$ ,  $x \rightarrow a$ .
- 59] Determine whether a function is continuous at a point.
- 60] Graph discontinuous functions.
- 61] Understand the intuitive concepts of limits of functions.
- 62] Graph rational and algebraic functions, including holes and asymptotes.
- 63] Find derivatives of functions by definition.
- 64] Find the derivative of a polynomial type function mentally and use it to find tangent line equations.
- 65] Sketch the graphs of functions using derivatives.
- 66] Solve maximum/minimum word problems using calculus.

- 67] Find instantaneous velocity and acceleration.
- 68] Prove theorems from geometry by using coordinates.
- 69] Find the equations of conic sections given geometric information.
- 70] Graph conic sections with center at  $(h,k)$ .
- 71] Solve systems of second degree equations.
- 72] Compute the eccentricity and latus rectum of conic sections.
- 73] State and explain the equation for the general quadratic relation.
- 74] Graph rotated conic sections by using technology.
- 75] Graph rotated conic sections by eliminating the “xy term.”

### **Assessment**

Students are generally assessed by in-class tests and occasional quizzes, which are administered regularly throughout a marking period. The students’ attitude, effort, and quality of homework preparation will also impact their term grade to a small degree. Teachers informally assess students every day by asking pivotal questions, as well as questions involving mechanics or concepts, and the students’ term grades may be positively affected to a small degree based on their responses.

A standardized midyear examination and final examination are administered to all students in this course in order to assess their long-term retention of the course material.

### **Technology Learning Objectives Addressed in This Course**

**(This section is for faculty and administrative reference; students and parents may disregard.)**

Course activity: skills &/or topics taught

- 1] Graphing calculators are integrated throughout to find extreme values of functions.
- 2] Graphing calculators are used in the Trigonometry portion to fit data to sinusoids.
- 3] Graphing calculators are used to aid in Matrix computations.
- 4] Precalculus Plotter Plus software is used where feasible.

### **Materials and Resources**

Text: Advanced Mathematics – Precalculus with Discrete Mathematics and Data Analysis. Brown, 2010.

Teachers use other texts for supplementary ideas, such as the Harcourt Brace Jovanovich text used previously and the text Precalculus – A Graphing Approach by Demara and Waits. Review materials that match both departmental examinations are used by all teachers of the course. Some teachers may employ the software package “Precalculus Plotter Plus” in the Mac Lab.